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Double torsion testing of prescription lenses

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In previous work, a modified double torsion test was developed to measure the fracture toughness of polymeric ophthalmic lenses with parallel surfaces. The method used production lenses so that the toughness of the manufactured articles could be measured directly. This method has now been extended to measure the K_{1c} of powered lenses in which the thickness of the sample varied with both X and Y coordinates instead of being constant. A computer model of the sample's breaking load was constructed and the predicted values from this were compared to experimental results and a good correlation was obtained. In addition, the computer-generated correction factors were validated using the more rigorous compliance curve technique.

1. Introduction

Ophthalmic lenses are lenses intended for use in spectacles. Those used in this research consisted of a thin-walled, shallow, spherical shell made of a rigid thermosetting polymer. In previous work [1] the fracture toughness, K_{1c}, was measured for the simple case of plano lenses that had parallel surfaces, and so were optically neutral. A method was developed using a modified double torsion technique which involved cutting the lenses in half and introducing a precracked notch in the resulting flat edge, Fig. 1. This specimen geometry was similar to the standard double torsion sample and gave a similar expression for K_{1c}, namely:

$$K_{1c} = P W_m \sqrt{\frac{3(1-\nu)}{W t^3 t_n}} \qquad (1)$$

where P is the load at fracture, W_m is the length of the moment arm, ν is Poisson's ratio, W is the width of the sample at the crack tip, t is the thickness of the torsion arms, and t_n is the thickness of the sample along the crack path. These dimensions are shown in Fig. 1.

The measured K_{1c} was independent of the precrack length provided the precrack length remained between certain limits. In addition, it was found that the curved shape of the lens caused negligible effect on the measured K_{1c}.

Most prescription lenses, however, have a convergent or divergent power and consequently have non-constant thicknesses. Thus, a crack growing from the notch in a modified double torsion sample would experience an increase or decrease in t_n depending on the individual lens prescription. This paper extends the previous work on plano lenses to cover the more complex geometry of the powered lens.

There are a number of methods which could be used to measure the fracture toughness of powered lenses. Classical fracture mechanics relates the critical strain energy release rate, G_{1c}, to the change in compliance with crack length, dC/da, and the load at fracture [2],

$$G_{1c} = \frac{P^2}{2t_n}\frac{dC}{da} \qquad (2)$$

Thus a direct measure of the fracture toughness could be made by measuring the change in compliance of the powered lens samples with increasing crack length.

Alternatively the change in compliance with crack length could be calculated using a mathematical analysis of the lens geometry. A third approach woud be to find empirical correction factors for the different powered lenses by comparing the measured fracture toughness values for the powered lenses using the modified double torsion specimens with the known K_{1c} for the same material obtained from conventional methods.

These three methods of measuring the fracture toughness of powered lenses are examined here. Each method has its advantages and disadvantages and hence needed to be assessed in light of the desired final use. The work was undertaken to find methods for testing lenses selected from production batches. In a commercial environment, the more rigorous approach of the compliance curve may not be the most appropriate method if quicker and cheaper methods can be found that have sufficient accuracy and product differentiation.

2. Theory

The double torsion test is based on Equation 2. With a prescription lens the value of t changes with the distance from the centre of the lens. This changing value affects Equation 2 in two ways. Not only does the value of t_n in the formula vary with crack length, but the value of dC/da is also complicated by the non-constant cross-section of the torsion arms. Hence Equation 1, previously used for the modified double torsion tests on plano lenses, has to be changed to allow for the above effects.

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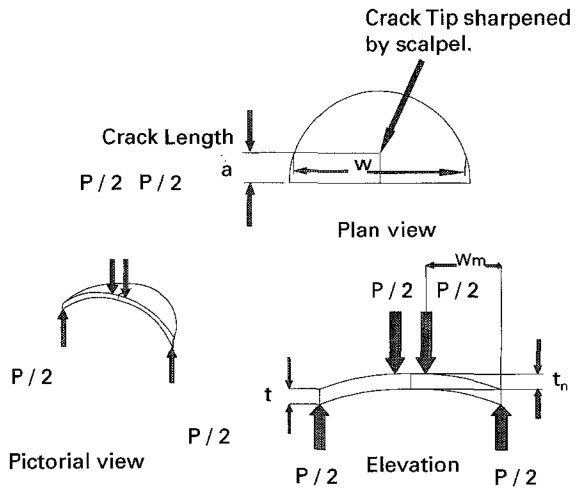

Figure 1 The modified double torsion sample.

2.1. Compliance curve method

The rigorous approach to measuring the fracture toughness is to evaluate the term $\mathrm{d}C/\mathrm{d}a$ experimentally. This requires measuring the compliance of powered lenses with different crack lengths and differentiating to obtain $\mathrm{d}C/\mathrm{d}a$. The fracture toughness can then be calculated from the fracture load P via Equation 2 and the critical stress intensity factor calculated by the relationship

$$K_{1c} = \sqrt{G_{1c} E^*} \tag{3}$$

where $E^* = E$, the elastic modulus, when the sample is in plane stress, and $E^* = E/(1 - \nu^2)$ for plane strain [2].

This method of measurement requires a series of lenses of varying crack length to be tested for each type and power of lens since $\mathrm{d}C/\mathrm{d}a$ would be specific to a given material and lens shape. This can be expensive in both time and materials.

2.2. Computer analysis of stress intensity in the sample

A more convenient approach would be to calculate from elasticity theory the value of $\mathrm{d}C/\mathrm{d}a$ for any powered lens.

To obtain the correct formula for the compliance of the torsion arms, the cross-section of the arms was first approximated by a trapezoid. The analytical solution to the torsional compliance of a trapezoid is [3]:

$$\frac{\Theta}{T} = \frac{a}{(k\,G)} \tag{4}$$

where Θ is the degree of twist in the torsion arm, T is the applied torque, a is the length of the torsion arm, G is the material shear modulus and;

$$k = \frac{1}{12} W_{\mathrm{m}} (m + n)(m^2 + n^2) - V_1 m^4 - V_s n^4$$

$$V_1 = 0.10504 - 0.10s + 0.0848s^2 - 0.06746s^3 + 0.0515s^4$$

$$V_s = 0.10504 + 0.10s + 0.0848s^2 + 0.06746s^3 + 0.0515s^4$$

$$s = \frac{(m - n)}{W_{\mathrm{m}}}$$

where m, n and W_{m} are as shown in Fig. 2. Θ/T is the torsional compliance, related to the linear compliance by $C = (\Theta/T)\, W_{\mathrm{m}}^2$.

The model calculates the compliance $\mathrm{d}C$ of the section of torsion arm that appears when the crack grows by $\mathrm{d}a$. Hence the rate of change $\mathrm{d}C/\mathrm{d}a$ is known, and the fracture toughness can be calculated from the load at fracture.

This model originally produced very good results for the -4.00, 0 and $+4.00$ lenses, but when the equations were tried over the full range of lens power from -4.00 to $+4.00$ the trapezoidal approximation was found to lack the desired degree of accuracy. Hence the model was rewritten to include a more accurate calculation of the torsional stiffness of the torsion arm cross-section, namely [3]:

$$k = \frac{1}{3} \frac{F}{\left(1 + \dfrac{4}{3} \dfrac{F}{A\, W_{\mathrm{m}}^2}\right)} \tag{5}$$

where;

$$F = \int_0^{W_{\mathrm{m}}} t^3\, \mathrm{d}W$$

A = area of cross-section and t = thickness of lens at a given point.

Using these equations the value of $\mathrm{d}C/\mathrm{d}a$ was then calculated numerically using a commercially available program, TK Solver [4]. Because the thickness varies with distance from the centre, t, b and t_n are all a function of crack length and the computer program had to calculate the instantaneous values for each of these variables to obtain the final result. This was done by representing each variable by a polynomial approximating the average dimensions of several examples of that specific type of lens. It was then possible to solve for $\mathrm{d}C/\mathrm{d}a$ at a given value of a, and

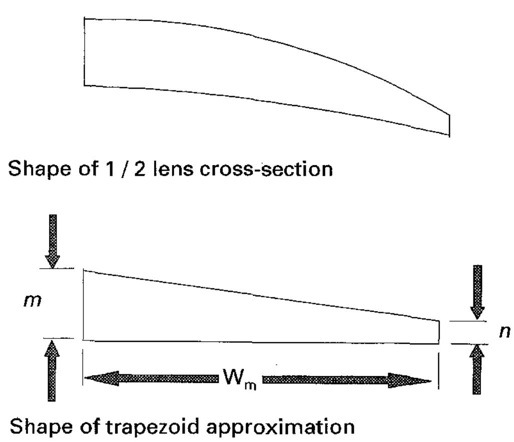

Figure 2 Torsion arm cross-section and the trapezoidal approximation showing the dimensions.

knowing the load at fracture for that crack length, G_{1c} was obtained. Using Equation 3, K_{1c} was calculated.

2.3. Empirical correction factor

Equation 1 takes the form of the load at fracture, P, multiplied by a variable $\{W_m\sqrt{(3(1+\nu)/(Wt^3t_n))}\}$ that depends on the geometry of the lens. For prescription lenses this 'geometry factor' would depend on both the size of the lens (W_m, W and t_n) and on its power. An alternative approach to obtaining the geometry factor for a powered lens would be to develop a correction factor which, when multiplied by the geometry factor for a plano lens of the same diameter and centre thickness, would give the correct geometry factor for the powered lens. In this work, the correction factor was determined empirically by measuring the apparent K_{1c} of a powered lens made from a material with a known K_{1c}. The correction factor required to give the known K_{1c} was then easily calculated.

3. Experimental procedures

3.1. Materials

The resin used in this study was a thermosetting resin supplied by Sola International Holdings Ltd. The resin was designated R1 and was a diallyl diethylene glycol carbonate.

The tests on plano lenses were performed on lenses which had an 83 mm radius of curvature, a diameter of 65 mm and a thickness of 3 mm. Further tests were done using prescription lenses with an 83 mm radius of curvature, a diameter of 70 mm and thicknesses which varied between 1.5 and 7 mm. The power of these lenses, which is the inverse of the focal length in metres, ranged from -4.00 to $+4.00$ with a negative value indicating a concave lens, and positive a convex one.

3.2. The modified double torsion method

The testing of the powered and plano lenses in this work was done using the modified double torsion test depicted in Fig. 1 and described in earlier work [1]. The point loads were applied by a compression cage mounted in a Model 1026 Instron testing machine. The crosshead speed used was 10 mm/min and all tests were carried out at 25 °C.

A graph of load versus time was obtained, allowing the calculation of fracture loads as well as sample compliance.

3.3. Compliance curve method

A series of $+4.00$ lenses were tested using the compliance curve method. These lenses were the most powerful of the convex, or magnifying, lenses with thicknesses ranging from 6.26 mm at the centre to 1.05 mm at the circumference. These lenses were made into modified double torsion samples with a range of different crack lengths. The crack lengths were spaced at 4 mm intervals from 0 to 20 mm, and five samples were tested at each crack length. The compliances of these samples were measured and plotted against crack length to obtain a graph from which dC/da could be measured.

Using Equations 2 and 3, K_{1c} was calculated and compared with the value obtained using the other methods.

3.4. Empirical correction factor

To determine the value of the correction factors a number of R1 lenses of powers ranging between -4.00 and $+4.00$ were tested and the loads at fracture recorded. These loads were multiplied by the geometry factor for a plano lens of the same centre thickness to give an apparent fracture toughness. The known fracture toughness was divided by the calculated toughness to give the required correction factor.

4. Results and discussion

4.1. Compliance curve

Fig. 3 shows the measured compliance against crack length for $+4.00$ powered lenses. It should be noted that with 20 mm crack lengths the samples failed at such low loads that accurate compliance measurements could not be made, hence this point was ignored. A cubic spline curve was fitted to the points, from which the gradient dC/da was obtained.

Using this value of dC/da, the value of G_{1c} could be calculated from Equation 2. If a was taken to be 10 mm, then $dC/da = 6\times10^{-4}\ \mathrm{N^{-1}}$, $P = 51.05$ N and $t_n = 5.94$ mm, and G_{1c} was found to be $130\ \mathrm{J\,m^{-2}}$. Repeating the calculation for different crack lengths gave similar values with an average of $120\ \mathrm{J\,m^{-2}}$ and a standard deviation of $10\ \mathrm{J\,m^{-2}}$.

The previously obtained value [1] of K_{1c} for R1 resin was $462\ \mathrm{kPa\sqrt{m}}$ with a standard deviation of 4%. Three-point bend tests on resin R1 gave a value of the modulus, E, as 1.3 GPa. Using this value and an estimate of ν for R1 of 0.35 the previously obtained value for K_{1c} of R1 resin of $462\ \mathrm{kPa\sqrt{m}}$ is equivalent to a G_{1c} of $144\ \mathrm{J\,m^{-2}}$ with a standard deviation of $12\ \mathrm{J\,m^{-2}}$. The difference between these two figures

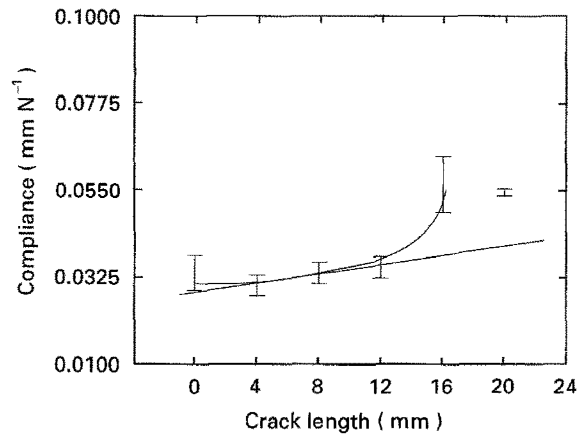

Figure 3 Plot of the compliance of a $+400$ power lens, showing a curve of best fit and the tangent at $a = 12$ mm.

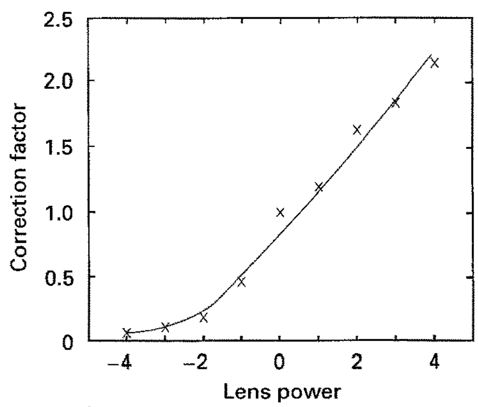

Figure 4 Empirical K_{1c} correction factor versus lens power for R1.

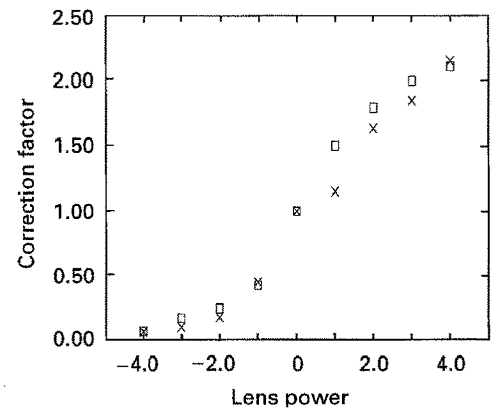

Figure 5 A comparison between the empirical (×) and the calculated (□) K_{1c} correction factors.

TABLE I Theoretical and experimental correction factors

Lens power	Theoretical factor	Experimental result	Percentage difference
+ 4.00	2.13	2.15	− 1
+ 3.00	1.569	1.85	− 15
+ 2.00	1.398	1.63	− 14.3
+ 1.00	1.261	1.15	− 5.7
0.00	1.00	1.00	0
− 1.00	0.377	0.44	− 14.8
− 2.00	0.195	0.7	+ 6.6
− 3.00	0.122	0.10	+ 15.5
− 4.00	0.067	0.6	+ 2.6

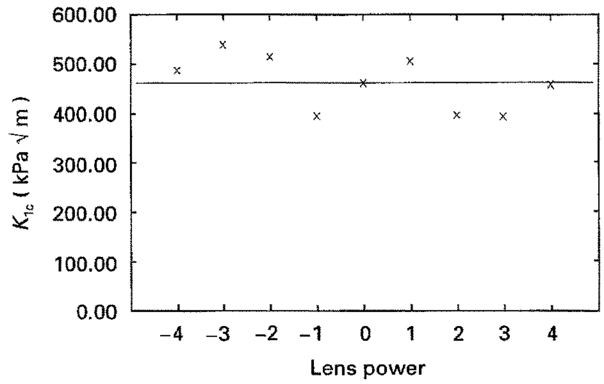

Figure 6 Fracture toughnesses calculated from the computer model and compared to the accepted value for this material of 462 kPa$\sqrt{m}$.

was within the accuracy expected in determining the value of G_{1c}. The value of 144 J m^{-2} was then used as the reference toughness for the other two methods.

The compliance curve method uses theoretically the most rigorous way of obtaining the fracture toughness of the lenses and gave results consistent with previous work. However, the amount of measurement and curve fitting that was required resulted in a large opportunity for error and was expensive in time and specimens. A simpler and quicker method was thus needed for routine measurement of the fracture toughness of these lenses.

4.2. Empirical correction factor

Since the fracture toughness of a material is independent of sample geometry, it was a straightforward exercise to obtain a correction factor for each lens power. The known value of K_{1c} for R1 was divided by the measured value of K_{1c} from a double torsion test of the powered lenses to give the correction factors. These experimental correction factors are plotted against the lens power in Fig. 4 and tabulated in Table I. Using Fig. 4 the correction factor for any lens within the − 4.0 to + 4.0 range can be determined, and hence the toughness measured using the modified double torsion test.

4.3. Theoretical correction factors

The correction factors obtained from the computer analysis are also listed in Table I and displayed graphically in Fig. 5.

The agreement between the computer model and the empirical correction factors was reasonably good, with the maximum discrepancies being approximately 15% for both moderately powered concave and convex lenses (+ 3 and − 3). Errors of this magnitude could be expected. The dimensions of the lenses were fitted to polynomials only to within 5% accuracy because of variation between the lenses, variation across a lens, etc. Hence the final correction factor, which was proportional to these measurements cubed, could not be expected to be more accurate than about 15%.

Using the computer-generated correction factors, the fracture toughness was calculated for different lens powers as shown in Fig. 6. Also shown in Fig. 6 is the previously obtained value [1] of K_{1c} for this material of 462 kPa$\sqrt{m}$. As can be seen there is good agreement between the current measurements and the previously obtained value.

5. Conclusions

The fracture toughness of powered lenses was measured using three different methods: the compliance curve approach, the use of a mathematical model to calculate K_{1c} for a given modified double torsion sample, and the use of empirical correction factors.

The fracture toughness measured for powered lenses made from the R1 resin using the compliance curve approach was in good agreement with previous measurements of fracture toughness for plano lenses and flat plates of this material.

Of these three methods, the compliance curve approach was the most rigorous, but was also the most expensive in terms of time and materials. A more convenient approach was to use correction factors. These can be found empirically but this required a number of lenses of the same shape made from a material with known K_{1c}. Alternatively the correction factors for powered lenses can be obtained directly from elasticity theory provided equations relating the inner and outer curvature of the lenses are available. The computer modelling method also correlated well with the other methods with results to within 15% of the other techniques.

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